

Validity of Thermodynamical Laws in Dark Energy Filled Universe

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We have considered the flat FRW model of the universe which is filled with only dark energy. The general descriptions of first and second laws of thermodynamics are investigated on the apparent horizon and event horizon of the universe. We have assumed the equation of state of three different types of dark energy models. We have examined the validity of first and second laws of thermodynamics on apparent and event horizons for these dark energies. For these dark energy models, it has been found that on the apparent horizon, first and second laws are always valid. On the event horizon, the laws are break down for dark energy models 1 and 2. For model 3, first law cannot be satisfied on the event horizon, but second law may be satisfied at the late stage of the evolution of the universe and so the validity of second law on the event horizon depends on the values of the parameters only.

I. INTRODUCTION

Recent observation [1, 2] of the luminosity of type Ia supernovae Wilkinson Microwave Anisotropy Probe, Sloan Digital Sky Survey, etc. indicate an accelerated expansion of the universe and the surveys of clusters of galaxies show that the density of matter is very much less than the critical density. This observation leads to a new type of matter which violate the strong energy condition. The matter content responsible for such a condition to be satisfied at a certain stage of evaluation of the universe is referred to as dark energy [3-6]. Most of the dark energy models involve one or more scalar fields with various actions and with or without a scalar field potential [7]. Now, as the observational data permits us to have a rather time varying equation of state, there are a bunch of models characterized by different scalar fields such as a slowly rolling scalar field, K-essence, tachyonic field, Chaplygin gas, a phantom model [8-12] etc. In a phantom model, we have the equation of state as $p = w\rho$, where $w < -1$. The simplest type of phantom model is a scalar field having a potential $V(\phi)$ with negative kinetic energy [13]. The condition $w = -1$ is named as the phantom divide. There are even models which can smoothly cross this phantom divide [14]. Currently constrained by the distance measurements of the type Ia supernova, and the current observational data constrain the range of equation of state as $-1.38 < w < -0.82$ [15].

The discovery of Hawking radiation of black holes in the semi-classical description black hole behaves like a black body and there is emission of thermal radiation. The temperature of a black hole is proportional to its surface gravity, and the entropy is also proportional to its surface area [16]. The Hawking temperature is given as $T = \frac{\kappa}{2\pi}$, where κ is the surface gravity of a black hole and the entropy of a black hole is $S = \frac{A}{4G}$, where A is its surface area. At the apparent horizon, the first law of thermodynamics is shown to be equivalent to Friedmann equations [17] if one takes the Hawking temperature and the entropy on the apparent horizon and the generalized second law of thermodynamics (GSLT) is obeyed at the horizon. If our Universe can be considered as thermodynamical system [18], the thermodynamical properties of the black hole can be generalized to space-time enveloped by the apparent horizon or the event horizon. The thermodynamical properties of the Universe may be similar to those of the black hole. Several authors [19] investigated the properties thermodynamical laws for different dark energy models.

In this work, the general descriptions of first law and GSL of thermodynamics (using Gibb's law) are investigated on the apparent horizon and event horizon of the universe. Then we investigate the thermodynamical properties of an accelerated expanding universe driven by dark energy with the variable equation of states of the form $p = w(z)\rho$. Recently, Zhang et al [20] have examined the validity of first law and generalized second law of thermodynamics for $w = w_0 + w_1 z$. Here we discuss the thermodynamic behavior by considering another three different parametrizations, describing the well known dark energy components: (i) $w(z) = w_0 + w_1 \frac{z}{1+z}$ [21]; (ii) $w(z) = -1 + \frac{(1+z)}{3} \frac{A_1 + 2A_2(1+z)}{A_0 + 2A_1(1+z) + A_2(1+z)^2}$ [22] and (iii) $w(z) = w_0 + w_1 \log(1+z)$ [23]. These choices of

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$w(z)$ are recently shown to be in good agreement with current observations in different ranges of z . The main motivation of the present work is that the first law and the GSL of thermodynamics are valid or not in the dark energy filled universe which is bounded by the apparent and event horizons. In the three types of well known dark energy models, we have examined the validity of the thermodynamical laws in diagrammatically. Finally we have drawn some interesting conclusions.

II. VALIDITY OF LAWS OF THERMODYNAMICS IN DIFFERENT DARK ENERGY MODELS

The Einstein field equations for homogeneous, isotropic and flat FRW universe are given by

$$H^2 = \frac{8\pi\rho}{3} \quad (1)$$

and

$$\dot{H} = -4\pi(\rho + p) \quad (2)$$

where $H(= \frac{\dot{a}}{a})$ is the Hubble parameter (choosing $G = c = 1$). The density ρ and pressure p of the dark energy are connected by the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0 \quad (3)$$

If $w = p/\rho$ be the equation of state of the dark energy then the above equation can be written as

$$\dot{\rho} + 3H(1 + w)\rho = 0 \quad (4)$$

Here we'll first present the general discussions of the conditions of the validity of first law and GSL of thermodynamics on the apparent and event horizons (which is also presented in our previous works [24]). Next, we examine the laws are valid or not on the apparent and event horizons for different dark energy models.

Now we consider the FRW universe as a thermodynamical system with the horizon surface as a boundary of the system. To study the generalized second law (GSL) of thermodynamics through the universe we deduce the expression for normal entropy using the Gibb's law of thermodynamics [25]

$$T_X dS_I = pdV + d(E_X) \quad (5)$$

where, S_I , p , V and E_X are respectively entropy, pressure, volume and internal energy within the apparent/event horizon and T_X is the temperature on the apparent horizon ($X = A$)/event horizon ($X = E$). Here the expression for internal energy can be written as $E_X = \rho V$. Now the volume of the sphere is $V = \frac{4}{3}\pi R_X^3$, where R_X is the radius of the apparent horizon (R_A)/event horizon (R_E) defined by [25]

$$R_A = \frac{1}{H} \quad (6)$$

and

$$R_E = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{a^2 H} \quad (7)$$

which immediately gives

$$\dot{R}_E = HR_E - 1 \quad (8)$$

The temperature and the entropy on the apparent/event horizon are

$$T_X = \frac{1}{2\pi R_X} \quad (9)$$

and

$$S_X = \pi R_X^2 \quad (10)$$

The amount of the energy crossing on the apparent/event horizon is [26]

$$-dE_X = 4\pi R_X^3 H T_{\mu\nu} k^\mu k^\nu dt = 4\pi R_X^3 H(\rho + p)dt = -H \dot{H} R_X^3 dt \quad (11)$$

The first law of thermodynamics on the apparent/event horizon is defined as follows:

$$-dE_X = T_X dS_X \quad (12)$$

Rate of change of internal entropy and total entropy are obtained as

$$\dot{S}_I = \frac{\dot{H} R_X^2 (H R_X - \dot{R}_X)}{T_X} \quad (13)$$

and

$$\dot{S}_I + \dot{S}_X = 2\pi R_X [\dot{H} R_X^2 (H R_X - \dot{R}_X) + \dot{R}_X] \quad (14)$$

The generalized second law states that total entropy can not be decreased i.e.,

$$\dot{S}_I + \dot{S}_X \geq 0 \text{ i.e., } \dot{H} R_X^2 (H R_X - \dot{R}_X) + \dot{R}_X \geq 0 \quad (15)$$

In our thermodynamic analysis, we are particularly interested in the parametrizations for the variation of dark energy equation of state w with redshift z as described below. We discuss the thermodynamic behavior by considering three different parametrizations, describing the dark energy component:

- **Model I:** $w = w_0 + w_1 \frac{z}{1+z}$, where w_0 and w_1 are constants [21] and $z = \frac{1}{a} - 1$ is the redshift.

In this case, dark energy density can be expressed as (from eq. (4))

$$\rho = \rho_0 (1+z)^{3(1+w_0+w_1)} e^{-\frac{3w_1 z}{1+z}} \quad (16)$$

and the field equation (1) reduces to

$$H^2 = H_0^2 (1+z)^{3(1+w_0+w_1)} e^{-\frac{3w_1 z}{1+z}} \quad (17)$$

where $H_0 = \sqrt{\frac{8\pi}{3}\rho_0}$ = present value of the Hubble parameter (at $a = 1$ i.e., $z = 0$).

From eq. (11), we have the amount of the energy crossing on the apparent horizon as

$$-dE_A = -H \dot{H} R_A^3 dt = -\frac{3}{2} H_0^{-1} (1+z)^{-\frac{3}{2}(1+w_0+w_1)} e^{\frac{3w_1 z}{1+z}} \left(1 + w_0 + \frac{w_1 z}{1+z}\right) \frac{dz}{1+z} \quad (18)$$

The temperature and entropy of the apparent horizon are

$$T_A = \frac{H}{2\pi} \quad (19)$$

and

$$S_A = \pi R_A^2 = \frac{\pi}{H^2} \quad (20)$$

So we have

$$T_A dS_A = -\frac{3}{2} H_0^{-1} (1+z)^{-\frac{3}{2}(1+w_0+w_1)} e^{\frac{3w_1 z}{2(1+z)}} \left(1 + w_0 + \frac{w_1 z}{1+z}\right) \frac{dz}{1+z} \quad (21)$$

Thus from equations (18) and (21), we have obtained

$$-dE_A = T_A dS_A \quad (22)$$

Therefore in the above model, the first law of thermodynamics is confirmed on apparent horizon.

From equation (14), we get

$$\frac{d(S_I + S_A)}{da} = \frac{d(S_I + S_A)}{dz} \frac{dz}{da} = \frac{9\pi}{2} H_0^{-2} (1+z)^{-4-3(w_0+w_1)} e^{-\frac{3w_1 z}{1+z}} [(1+w_0)(1+z) + w_1 z]^2 \geq 0 \quad (23)$$

So, in this model second law is always valid on the apparent horizon.

In this model, the radius of the event horizon in terms of redshift z can be written as

$$R_E = -\frac{1}{1+z} \int_z^{-1} \frac{dz}{H} = -\frac{1}{1+z} H_0^{-1} \int_z^{-1} (1+z)^{-\frac{3}{2}(w_0+w_1)} e^{\frac{3w_1 z}{2(1+z)}} dz \quad (24)$$

From eq. (11), we have the amount of the energy crossing on the event horizon as

$$-dE_E = \frac{3}{2} R_E^3 H^2 \left(1 + w_0 + \frac{w_1 z}{1+z}\right) \frac{dz}{1+z} \quad (25)$$

Using equations (9) and (10), we get

$$T_E dS_E = dR_E = \frac{(1-HR_E)dz}{(1+z)H} \quad (26)$$

From (26) and (27), we have seen that $dE_E + T_E dS_E$ is a function of z . In fig.1, we have drawn $dE_E + T_E dS_E$ against redshift z for $w_0 = -1.2$ and $w_1 = .98$. It has been seen that the curve $dE_E + T_E dS_E$ is not coincide with the z axis, i.e.,

$$dE_E + T_E dS_E \neq 0, \text{ i.e., } -dE_E \neq T_E dS_E \quad (27)$$

So first law does not valid on the event horizon.

Now, from eq. (14), we obtain

$$\frac{d(S_I + S_E)}{da} = \frac{d(S_I + S_E)}{dz} \frac{dz}{da} = -(1+z)^2 \pi \left[2R_E^4 H \frac{dH}{dz} + \left\{ 3R_E^3 H^2 \left(1 + w_0 + w_1 \frac{z}{1+z}\right) + 2R_E \right\} \frac{dR_E}{dz} \right] \quad (28)$$

which is a complicated function of redshift z . In fig.2, we have drawn $(dS_I + dS_E)$ against redshift z for $w_0 = -1.2$ and $w_1 = .98$. From figure, we have seen that

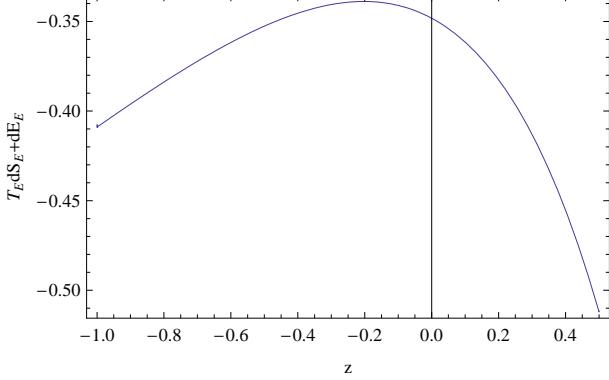


Fig.1

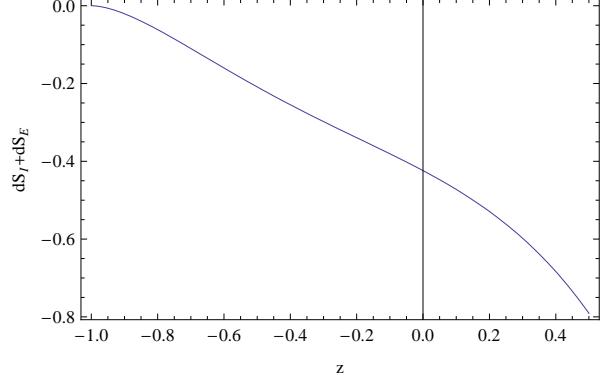


Fig.2

Fig. 1 and 2 show the variations of $(dE_E + T_E dS_E)$ and $(dS_I + dS_E)$ respectively against redshift z for $w_0 = -1.2$ and $w_1 = .98$ in model 1.

$$\frac{d(S_I + S_E)}{da} < 0 \quad (29)$$

That means second law of thermodynamics does not hold on the event horizon of the universe.

• **Model 2:** $w(z) = -1 + \frac{(1+z)}{3} \frac{A_1+2A_2(1+z)}{A_0+2A_1(1+z)+A_2(1+z)^2}$, where A_0 , A_1 and A_2 are constants [22].

This ansatz is exactly the cosmological constant $w = -1$ for $A_1 = A_2 = 0$ and DE models with $w = -2/3$ for $A_0 = A_2 = 0$ and $w = -1/3$ for $A_0 = A_1 = 0$. It has also been found to give excellent results for DE models in which the equation of state varies with time including quintessence, Chaplygin gas, etc.

In this model, equation (1) can be expressed in the form:

$$H^2 = \frac{H_0^2}{A_0 + A_1 + A_2} [A_0 + A_1(1+z) + A_2(1+z)^2] \quad (30)$$

Using equations (9) - (11), we have (on apparent horizon)

$$-dE_A = -\frac{1}{2} H_0^{-1} \sqrt{A_0 + A_1 + A_2} \frac{[A_1 + 2A_2(1+z)]}{[A_0 + A_1(1+z) + A_2(1+z)^2]^{\frac{3}{2}}} dz \quad (31)$$

and

$$T_A dS_A = dR_A = -\frac{1}{2} H_0^{-1} \sqrt{A_0 + A_1 + A_2} \frac{[A_1 + 2A_2(1+z)]}{[A_0 + A_1(1+z) + A_2(1+z)^2]^{\frac{3}{2}}} dz \quad (32)$$

From above expressions, we see that

$$-dE_A = T_A dS_A \quad (33)$$

Therefore, first law of thermodynamics is satisfied on apparent horizon of the universe.

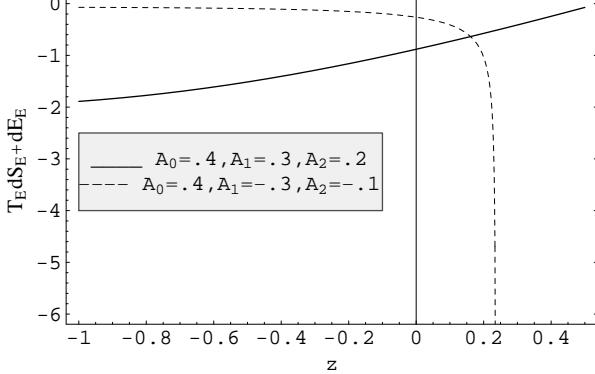


Fig.3

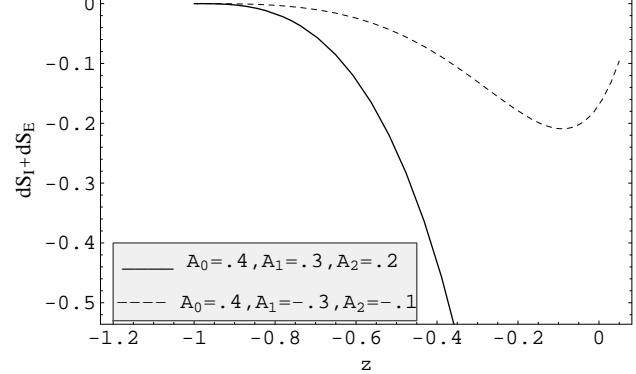


Fig.4

Fig. 3 and 4 show the variations of $(dE_E + T_E dS_E)$ and $(dS_I + dS_E)$ respectively against redshift z for different values of parameters of model 2.

From equation (14), we get (on apparent horizon)

$$\frac{d(S_I + S_A)}{da} = -\pi H_0^{-2}(1+z)^2 \frac{(A_0 + A_1 + A_2)[A_1 + 2A_2(1+z)]}{[A_0 + A_1(1+z) + A_2(1+z)^2]^2} \left[\frac{[A_1 + 2A_2(1+z)][2A_0 + A_1(1+z)]}{2[A_0 + A_1(1+z) + A_2(1+z)^2]} - 1 \right] \geq 0 \quad (34)$$

So, on the apparent horizon, second law is satisfied.

Using equations (9) - (11), we have (on event horizon)

$$-dE_E = 4\pi R_E^3 H \rho (1+w) dt = -\frac{3}{2} R_E^3 H^2 \frac{(1+w) dz}{1+z} \quad (35)$$

and

$$T_E dS_E = dR_E = -\frac{1}{H_0} \frac{\sqrt{A_0 + A_1 + A_2}}{\sqrt{A_0 + A_1(1+z) + A_2(1+z)^2}} dz \quad (36)$$

From (34) and (35), we have seen that $dE_E + T_E dS_E$ is a function of z . In fig.3, we have drawn $dE_E + T_E dS_E$ against redshift z for different values of parameters (like, $A_0 = .4, A_1 = .2, A_2 = .1$ and $A_0 = .4, A_1 = -.2, A_2 = -.1$). It has been seen that the curve $dE_E + T_E dS_E$ is not coincide with the z axis, i.e.,

$$dE_E + T_E dS_E \neq 0, \text{ i.e., } -dE_E \neq T_E dS_E \quad (37)$$

that is, first law does not valid on the event horizon. From equation (14), we get (on event horizon)

$$\frac{d(S_I + S_E)}{da} = -(1+z)^2 \pi \left[2R_E^4 H \frac{dH}{dz} + \left(\frac{R_E^3 H^2 (1+z)(A_1 + 2A_2(1+z))}{(A_0 + A_1(1+z) + A_2(1+z)^2)} + 2R_E \right) \frac{dR_E}{dz} \right] \quad (38)$$

which is a complicated function of z . In fig.4, we have drawn $(dS_I + dS_E)$ against redshift z for different values of the parameters like, $A_0 = .4, A_1 = .2, A_2 = .1$ and $A_0 = .4, A_1 = -.2, A_2 = -.1$. In all the cases, we have seen that

$$\frac{d(S_I + S_E)}{da} < 0 \quad (39)$$

That means second law of thermodynamics does not hold on the event horizon of the universe.

- **Model 3:** $w = w_0 + w_1 \log(1+z)$, where w_0 and w_1 are constants [23].

From equation (1), we get

$$H^2 = H_0^2 (1+z)^{3(1+w_0)} e^{\frac{3}{2}w_1[\log(1+z)]^2} \quad (40)$$

Using equations (9) - (11), we have (on apparent horizon)

$$-dE_A = 4\pi R_A^3 H \rho (1+w) dt = -\frac{3}{2} H_0^{-1} [1+w_0+w_1 \log(1+z)] (1+z)^{-\frac{3}{2}(1+w_0)-1} e^{-\frac{3}{4}w_1[\log(1+z)]^2} dz \quad (41)$$

and

$$T_A dS_A = dR_A = -\frac{3}{2} H_0^{-1} [1+w_0+w_1 \log(1+z)] (1+z)^{-\frac{3}{2}(1+w_0)-1} e^{-\frac{3}{4}w_1[\log(1+z)]^2} dz \quad (42)$$

Thus,

$$-dE_A = T_A dS_A \quad (43)$$

So, first law of thermodynamics is valid on the apparent horizon.

From equation (14), we get (on apparent horizon)

$$\frac{d(S_I + S_A)}{da} = \frac{d(S_I + S_A)}{dz} \frac{dz}{da} = \frac{9}{2} H_0^{-2} [1+w_0+w_1 \log(1+z)]^2 (1+z)^{-3(1+w_0)+1} e^{-\frac{3}{2}w_1[\log(1+z)]^2} \geq 0 \quad (44)$$

Therefore, the second law is still valid on the apparent horizon.

The radius of the event horizon in terms of redshift z can be written as

$$R_E = H_0^{-1} \left[e^{-\frac{3}{4}w_1[\log(1+z)]^2} (1+z)^{-\frac{3}{2}(1+w_0)-1} \left(-\frac{2}{1+3w_0} + \frac{2z}{2+3(1+w_0)} \right) \right] \quad (45)$$

Using equations (9) - (11), we have (on event horizon)

$$-dE_E = 4\pi R_E^3 H \rho (1+w) dt = -\frac{3}{2} R_E^3 [1+w_0+w_1 \log(1+z)] \frac{H^2}{1+z} dz \quad (46)$$

and

$$T_E dS_E = dR_E \quad (47)$$

From (45) and (46), we have seen that $dE_E + T_E dS_E$ is a complicated function of z . In fig.5, we have drawn $dE_E + T_E dS_E$ against redshift z . It has been seen that the curve $dE_E + T_E dS_E$ is not coincide with the z axis, i.e.,

$$dE_E + T_E dS_E \neq 0, \text{ i.e., } -dE_E \neq T_E dS_E \quad (48)$$

So the first law does not hold on the event horizon.

From equation (14), we get (on event horizon)

$$\frac{d(S_I + S_E)}{da} = -(1+z)^2 \pi \left[2R_E^4 H \frac{dH}{dz} + \{2R_E + 3R_E^2 H^2 (1+w_0+w_1 \log(1+z))\} \frac{dR_E}{dz} \right] \quad (49)$$

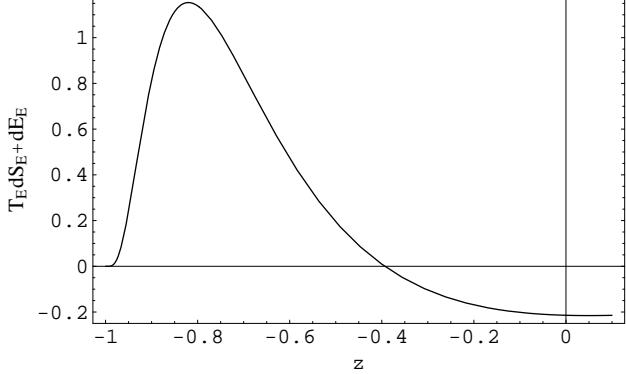


Fig.5

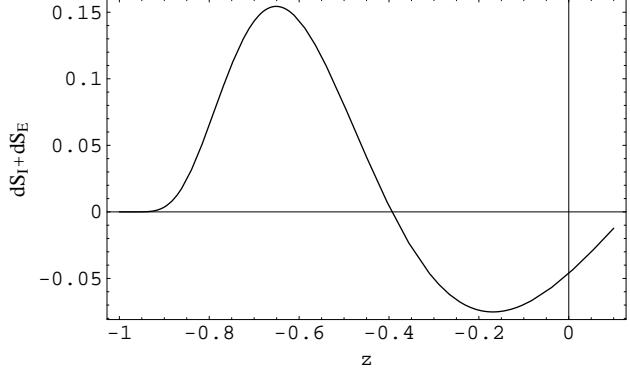


Fig.6

Fig. 5 and 6 show the variations of $(dE_E + T_E dS_E)$ and $(dS_I + dS_E)$ respectively against redshift z for $w_0 = -0.5$ and $w_1 = 1$ in model 3.

In fig.6, we have drawn $(dS_I + dS_E)$ against redshift z for $w_0 = -0.5$ and $w_1 = 1$. For these values of the parameters, the dark energy propagates from quintessence dominated era to phantom era. From graphical representation, it has been seen that $\frac{d(S_I + S_E)}{da}$ changes sign from negative to positive as z decreases. So it may be conclude that in quintessence dominated era second law does not valid on the event horizon but in phantom dominated era second law is valid on the event horizon.

III. DISCUSSIONS

We have considered the flat FRW model of the universe which is filled with only dark energy obeys equation of state $p = w\rho$. Here w is not a constant, it is assumed as a function of redshift z . The apparent horizon of the universe always exists and the thermodynamical properties related to the apparent horizon has been studied. We have investigated to an accelerated expanding universe driven by DE of time-dependent (i.e., redshift) equation of state. The event horizon and apparent horizon are in general different surfaces. The general descriptions of first and second laws of thermodynamics are investigated on the apparent horizon and event horizon of the universe. We have assumed the equation of state of different types of dark energy models. Here we discuss the thermodynamic behavior by considering three different parametrizations, describing the dark energy component: (1) Model 1: $w = w_0 + w_1 \frac{z}{1+z}$; (2) Model 2: $w(z) = -1 + \frac{(1+z)}{3} \frac{A_1 + 2A_2(1+z)}{A_0 + 2A_1(1+z) + A_2(1+z)^2}$; (3) Model 3: $w = w_0 + w_1 \log(1+z)$. These choices of w are recently shown to be in good agreement with current observations in different ranges of z .

In this work, we have tried to apply the usual definition of the temperature and entropy as that of the apparent horizon to the cosmological event horizon and examine the validity of first and the second laws of thermodynamics. For these dark energy models, it has been found analytically that on the apparent horizon, first and second laws are always valid. On the event horizon, we have found complicated expressions for these dark energy models. So analytically, we cannot draw any conclusions i.e., it is not possible to infer about the validity of the thermodynamical laws on the event horizon for these dark energy models. So graphical investigations have been needed to draw the final conclusions. From diagrams, we have been seen that on the event horizon the laws are break down for dark energy models 1 and 2. For model 3, first law cannot be satisfied on the event horizon and second law on the event horizon cannot be satisfied in the early stage of the universe, but it may be satisfied at the late stage of the evolution of the universe and so the validity of second law on the event horizon depends on the values of the parameters only.

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